PERGAMON

International Journal of Multiphase Flow 27 (2001) 1945–1964

International Journal of Multiphase Flow

www.elsevier.com/locate/ijmulflow

Particle–gas turbulence interactions in a kinetic theory approach to granular flows

Yonghao Zhang [a], Jason M. Reese [b,*]

[a] *Department of Engineering, University of Aberdeen, Aberdeen AB24 3UE, UK*

[b] *Department of Mechanical Engineering, King's College London, Strand, London WC2R 2LS, UK*

Received 11 December 2000; received in revised form 21 March 2001

Abstract

We describe a new two-fluid model for gas–solid flows that incorporates the gas turbulence influence on the random motion of the particles via a generalised kinetic theory, as well as a new gas turbulence modulation model. Simulation results for fully developed steady vertical pipe flows are in good quantitative agreement with experimental measurements. Our results show that the influence of gas turbulence on the particle microscopic flow behaviour cannot be ignored for relatively dilute flows, especially with smaller particles. Our new turbulent model captures the essential characteristics of gas turbulence modulation in the presence of particles, and can be solved for a wide range of particle–particle restitution coefficients. © 2001 Elsevier Science Ltd. All rights reserved.

Keywords: Gas–solid flows; Granular flows; Kinetic theory; Turbulence modulation; Two-fluid model

1. Introduction

Gas–solid two-phase flows are important in many engineering applications and industrial processes, e.g., materials-handling engineering, circulating fluidised beds, pneumatic conveying and nuclear reactor cooling. Also, they are relevant in various natural phenomena, e.g., sandstorms, moving sand dunes, cosmic dusts, snow avalanches, dust explosions and settlement. Understanding the physical mechanisms governing these flows is essential for the optimal design of these industrial processes, and for modelling these natural phenomena.

Numerical models for gas–solid flows play a key role in both fundamental research and engineering applications. There are two parallel paths for modelling particles flowing with a carrier

* Corresponding author. Tel.: +44-20-7848-2322; fax: +44-20-7848-2932.
E-mail address: jason.reese@kcl.ac.uk (J.M. Reese).

0301-9322/01/$ - see front matter © 2001 Elsevier Science Ltd. All rights reserved.
PII: S0301-9322(01)00039-8

gas, according to the way the particle phase is treated: the Lagrangian (or particle trajectory) approach, and the Eulerian (or two-fluid) approach. The Lagrangian approach, which includes distinct element and direct simulation methods, describes the motion of each solid particle by a separate equation. These methods have distinct advantages when considering flows with e.g. particles of different sizes, and yield detailed information on the particle flow but at the cost of long computational time and high computer memory demands. In two-fluid models, the particle phase is treated as a continuous medium so that the governing equations are in a similar form to the hydrodynamic equations. Appropriate averaged properties of the particle phase are required. The advantages of a two-fluid model are: it requires less computational effort, it is more suitable for engineering applications where no details of the individual particle motion are necessary, and it is perhaps the only practical way of approaching modelling denser flows.

Elghobashi (1994) classified gas–solid flows into two regimes: dilute and dense flows. In the dilute regime, where the solid volume fraction is less than 0.1%, the collisions between particles have negligible effect on the carrier gas turbulence. In the dense regime, where the solid volume fraction is greater than 0.1%, the collisions begin to play an important role in the flows. This so-called "four-way coupling effect" should be considered.

On the other hand, when the solid volume fraction is very high, collisions between particles dominate the flows, and the carrier phase can be neglected. Many competing kinetic theories of granular flow (see, e.g., Jenkins and Savage, 1983; Lun et al., 1984; Jenkins and Richman, 1985; Abu-Zaid and Ahmadi, 1990; Gidaspow, 1994) have been proposed following the pioneering work of Bagnold (1954), Ogawa et al. (1980), and Savage and Jeffrey (1981). These theories have been reviewed by Savage (1984) and Campbell (1990).

The kinetic theory of granular flow has had some success in modelling the interaction mechanism between the mean and fluctuating solid velocities. Sinclair and Jackson (1989) incorporated this theory in a two-fluid model, where they assumed that the mechanism of the interaction between the mean and fluctuating particulate phase is the same as in the kinetic theory of granular flow alone. The interaction between the particulate phase and the gas phase was a mutual drag force, and the gas flow was assumed laminar. Although certain important physical mechanisms were not included, their results revealed many experimentally observed flow phenomena: co-current upflow, co-current downflow, counter-current flow, non-homogeneous radial distribution of solid concentration, etc. Since then, kinetic theories of granular flow have been widely used to model the solid phase motion in a multiple-phase flow system (e.g., Louge et al., 1991; Bolio and Sinclair, 1995; Bolio et al., 1995; Pita and Sundaresan, 1993; Nieuwland et al., 1996; Neri and Gidaspow, 2000; Mathiesen et al., 2000).

In the regime where the solid volume fraction is greater than 0.1% (so that the collisions between particles should be considered, but not dense enough that the gas turbulence can be neglected), we may need to investigate how the interstitial turbulent gas affects the constitutive equations of the solid phase – which is always neglected in a kinetic theory of "dry" granular flow. Also, the question of how the presence of particles modulates the gas turbulence should be addressed. This paper examines these issues arising in this regime of solid volume fraction.

Louge et al. (1991) began to incorporate both the kinetic theory of dry granular flow and gas turbulence into a two-fluid model for relatively dilute flows. The gas turbulence was described by a one-equation turbulence model, and the same characteristic length scale of the turbulence as in pure gas flow was adopted. Yuan and Michaelides (1992) proposed a turbulence modification model by arguing that the wake is responsible for the augmentation of turbulence and the work done on the particles is responsible for the attenuation of turbulence. Their results are in good agreement with experimental data. Later, Yarin and Hetsroni (1994) proposed a more detailed expression for the wake. Bolio et al. (1995) extended the work of Louge et al. (1991) to a two-equation turbulent model: the k–ε eddy viscosity model. They found that the gas turbulence intensity was somewhat underestimated. Bolio and Sinclair (1995) improved the model by adopting the turbulence modification model of Yuan and Michaelides (1992) and confirmed that the wake is responsible for the enhancement of gas turbulence. Kenning and Crowe (1997) suggested that the turbulence enhancement may be associated with the work done by the particle drag, and the inter-particle spacing is an additional physical restriction to the flow which is responsible for the attenuation of gas turbulence. Crowe and Gillandt (1998), Crowe and Wang (2000) and Crowe (2000) derived and improved a detailed turbulent modulation model. These authors argued that the common approach to the derivation of the turbulent kinetic energy balance equation (e.g. Louge et al., 1991) treats the averaged velocity as if it were a local velocity in the momentum equations of both phases.

In a gas–solid flow, whether the flow is governed by gas turbulence or particle collisions can be classified by three characteristic time-scales: the characteristic time-scale of eddies, t_1^t, the mean particle relaxation time, t_{12}^x, and the mean particle collision time, t_2^c. If $t_{12}^x \ll t_1^t$, particle motion is controlled by the gas flow, and if $t_2^c \ll t_{12}^x$, the flow is governed by particle collisions. But, when the particle relaxation time is not much larger than the particle collision time, the random motion of the particles will be affected by the gas-phase turbulence. Peirano and Leckner (1998) derived a competing kinetic theory of granular flow with an interstitial gas, based on the work of Jenkins and Richman (1985), which incorporated the influence of the gas turbulence on the particle random motion. As a result, the gas turbulence appears in their constitutive equations for the solid phase. If the interstitial gas were omitted, their results are identical to the work of Jenkins and Richman (1985) and Lun et al. (1984). Although their model is still restricted to slightly inelastic particle–particle collisions, it removed the assumption in previous kinetic theories that the interstitial gas can be neglected.

Crowe and his colleagues' new turbulent modulation model needs to be investigated numerically, which is the first aim of this paper. Second, we will examine the influence of the interstitial turbulent gas on the random motion of particles in a relatively dilute flow regime. All the simulation results will be compared with the comprehensive experimental measurements of Tsuji et al. (1984) and Tsuji (1993) for fully developed gas–solid flows. Throughout we will assume that the fully developed turbulent gas–solid flow is steady and axi-symmetric. This may be an appropriate assumption for fully developed flow in the small-diameter pipes Tsuji and his colleagues examined, where the spatial inhomogeneity of the particulate phase flow structure is likely to be small. Finally, we will assess the sensitivity of our new model to the particle–particle restitution coefficient.

2. The mathematical model

We assume that particle–particle collisions are nearly elastic and treat the coefficients of restitution of particle–particle and particle–wall as constants in the collisions. The governing equations for a dispersed solid phase and a carrier gas phase are locally averaged. The momentum equations are given by Anderson and Jackson (1967)

Solid phase:

$$\varepsilon_2\rho_2\frac{\mathrm{D}}{\mathrm{D}t}\boldsymbol{U} = -\varepsilon_2\nabla P - \nabla\cdot\tau_2 + \overline{\boldsymbol{F}}_{\mathrm{drag}} + \varepsilon_2\rho_2\boldsymbol{g}, \tag{1}$$

Gas phase:

$$\varepsilon_1\rho_1\frac{\mathrm{D}}{\mathrm{D}t}\boldsymbol{V} = -\varepsilon_1\nabla P - \nabla\cdot\tau_1 - \overline{\boldsymbol{F}}_{\mathrm{drag}}, \tag{2}$$

where subscripts 1 and 2 represent the gas phase and solid phase, respectively, e.g., ε_1 and ε_2 are the volume fractions of the gas phase and the particle phase; ρ is the density; is the stress; $\boldsymbol{U}$ is the averaged velocity of the solid phase; $\boldsymbol{V}$ is the averaged velocity of the gas phase; $\overline{\boldsymbol{F}}_{\mathrm{drag}}$ is the averaged drag force; $\boldsymbol{g}$ is the gravity force.

2.1. The solid phase

For a two-phase flow, it is important to model the stresses of both phases and the inter-phase transfer property correctly. For the stress of the solid phase, we adopt the work of Peirano and Leckner (1998), viz.

$$\tau_{2ij} = (P_2 - \zeta_2 S_{kk})\delta_{ij} - 2\varepsilon_2\rho_2(v_2^{\mathrm{c}} + v_2^{\mathrm{t}})\hat{S}_{ij}, \tag{3}$$

where the particle pressure is

$$P_2 = \varepsilon_2\rho_2(1 + 2\varepsilon_2\chi(1+e))T, \tag{4}$$

the bulk viscosity is

$$\zeta_2 = \frac{4}{3}d\varepsilon_2^2\rho_2\chi(1+e)\sqrt{\frac{T}{\pi}}, \tag{5}$$

where T is the granular temperature, defined as $1/3\langle\boldsymbol{u}'\cdot\boldsymbol{u}'\rangle$; $\boldsymbol{u}'$ is the fluctuational velocity of the solid phase; d is the particle diameter; $S_{2ij} = (U_{i,j} + U_{j,i})/2$; and $\hat{S}_{2ij} = S_{2ij} - S_{2mm}\delta_{ij}/3$. Expressions for the collisional viscosity, v_2^{c}, and the turbulent viscosity, v_2^{t}, are given in Appendix A.

Because the stress depends on the turbulent kinetic energy, we also need a closure equation for the balance of the fluctuational energy

$$\frac{3}{2}\varepsilon_2\rho_2\frac{\mathrm{D}}{\mathrm{D}t}T = -\nabla\cdot\boldsymbol{q} - \tau_2 : \nabla\boldsymbol{U} - \beta_0(3T - k_{12}) + \boldsymbol{I}. \tag{6}$$

The expression for the fluctuational energy flux, $\boldsymbol{q}$, is given as in Peirano and Leckner (1998), viz.

$$\boldsymbol{q} = -\frac{3}{2}\varepsilon_2\rho_2(k_2^{\mathrm{t}} + k_2^{\mathrm{c}})\nabla T. \tag{7}$$

If there is no interstitial gas, the expressions for the particle stress and fluctuational energy flux are identical to Lun et al. (1984) and Jenkins and Richman (1985). Expressions for the collisional diffusion coefficient, k_2^{c}, and the turbulent diffusion coefficient, k_2^{t}, are given in Appendix A. Energy dissipation is described by

$$I = 12\varepsilon_2^2(e^2 - 1)\chi\rho_2 \frac{1}{d\sqrt{\pi}} T^{3/2}. \tag{8}$$

The effect of turbulence on the drag force on a particle still needs further investigation, especially for a particle flow in a cloud where the inter-particle effect is important (Crowe, 1997; Littman et al., 1996). The volume-averaged drag force can be given as (Zhang and Reese, 2000)

$$\overline{\mathbf{F}}_{\text{drag}} = \beta_0(\boldsymbol{V} - \boldsymbol{U}), \tag{9}$$

where the momentum transfer coefficient is

$$\beta_0 \approx \frac{3}{4} C_{\text{D}} \frac{(1-\varepsilon)\rho_1}{d} \varepsilon^{-2.65} U_{\text{r}}, \quad \varepsilon \geqslant 0.8 \tag{10}$$

and U_r is the mean slip velocity

$$U_{\text{r}} = \sqrt{(\boldsymbol{V} - \boldsymbol{U})^2 + \frac{8T}{\pi}} \tag{11}$$

and C_{D} is the drag coefficient given by

$$C_{\text{D}} = \left(0.28 + \frac{6}{\sqrt{Re_{\text{p}}}} + \frac{21}{Re_{\text{p}}}\right). \tag{12}$$

The particle Reynolds number Re_{p} is

$$Re_{\text{p}} = \frac{\varepsilon\rho_1 U_{\text{r}}}{\mu}, \tag{13}$$

where μ is the gas viscosity. The fluctuational energy transfer rate k_{12} will be discussed below.

2.2. The gas turbulence modulation

The gas effective stress has two parts: a viscosity stress and a Reynolds turbulent stress, i.e.

$$\tau_{1ij} = (\mu + \mu_{\text{t}})\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) - \frac{2}{3}(\mu + \mu_{\text{t}})\nabla \cdot v\delta_{ij} - \frac{2}{3}\rho_1 k_1 \delta_{ij}, \tag{14}$$

where μ_{t} is the eddy viscosity; the turbulent kinetic energy, k_1 is $1/2\langle \boldsymbol{v}' \cdot \boldsymbol{v}' \rangle$, and $\boldsymbol{v}'$ is the fluctuational velocity of gas. The exact equation for k_1 for a single fluid flow can be derived from the Navier–Stokes equations. For a gas–solid system, in a similar way to a single-phase flow, the commonly used expression for k_1 is given as (see, Chen and Wood, 1985; Berlemont et al., 1990; Louge et al., 1991)

$$\frac{\partial}{\partial t}(\varepsilon_1\rho_1 k_1) + \frac{\partial}{\partial x_j}(\varepsilon_1\rho_1 V_j k_1) = \frac{\partial}{\partial x_j}(\sigma_k \frac{\partial k_1}{\partial x_j}) - \varepsilon_1\rho_1\langle \boldsymbol{v}'\boldsymbol{v}' \rangle \frac{\partial}{\partial x_j} V_i + \beta_0(\langle \boldsymbol{u}' \cdot \boldsymbol{v}' \rangle - 2k_1) - \varepsilon_1\varepsilon, \tag{15}$$

where the turbulent kinetic energy dissipation rate, ε, is $\mu\langle(\nabla \boldsymbol{v}')^2\rangle$.

However, Crowe and Gillandt (1998) argued that the coupling term is not correctly modelled in Eq. (15) because the coupling drag force is an averaged value which cannot be regarded as a point value in the derivation of Eq. (14). Their model asserted

$$\frac{\partial}{\partial t}(\varepsilon_1\rho_1 k_1) + \frac{\partial}{\partial x_j}(\varepsilon_1\rho_1 V_j k_1) = \frac{\partial}{\partial x_j}\left(\sigma_k \frac{\partial k_1}{\partial x_j}\right) - \varepsilon_1\rho_1\langle \boldsymbol{v}'\boldsymbol{v}'\rangle \frac{\partial}{\partial x_j} V_i + \beta_0|\boldsymbol{U} - \boldsymbol{V}|^2 + \beta_0(3T - \langle \boldsymbol{u}'\cdot\boldsymbol{v}'\rangle) - \varepsilon_1\varepsilon. \quad (16)$$

The first term on the right-hand side of this equation is the diffusion of turbulent kinetic energy; the second term is the generation by gradients; the third term is the work done by the particle drag force which converts the kinetic energy of mean flow into turbulent kinetic energy; the fourth term represents the transfer of kinetic energy of the particle motion to turbulent kinetic energy of the carrier phase, which has been identified as four-way coupling – which is relatively small in a dilute flow (Elghobashi, 1994); the final term is the dissipation.

The eddy viscosity may be given as

$$\mu_t = C_1\rho_1\sqrt{k_1}\,l, \quad (17)$$

where C_1 is a constant parameter. Louge et al. (1991) adopted

$$\varepsilon = C_2 k_1^{3/2}/l, \quad (18)$$

where l is the turbulent dissipation length scale and C_2 is a constant. Peirano and Leckner (1998) gave the expression for σ_k as

$$\sigma_k = \varepsilon_1(\mu + \mu_t/Pr), \quad (19)$$

where Pr is the effective Prandtl number, which relates the eddy diffusion of turbulence kinetic energy to the momentum eddy viscosity. Louge et al. (1991) assumed $Pr = 1$ in their calculation.

In a one-equation turbulence model, $C_1C_2 \approx 0.08$ (Launder and Spalding, 1972). As Rodi (1993) pointed out, the individual values of the constant C_1 and C_2 are not important in a single gas turbulent flow, but only their product. However, these two constants can be used to estimate the length scale as

$$l \approx \left(\frac{C_2}{C_1^3}\right)^{1/4} l_m \quad (20)$$

where l_m is the mixing length of gas turbulence, which can be estimated as in Louge et al. (1991).

For a gas–solid flow, Kenning and Crowe (1997) proposed a modification of this viscosity dissipation length scale to account for the presence of particles

$$l_h = \frac{l\lambda}{l + \lambda}, \quad (21)$$

where λ is the inter-particle spacing which can be given as

$$\lambda \approx \left(\frac{\pi}{6\varepsilon_2}\right)^{1/3} d. \quad (22)$$

This approach regards the inter-particle space as an equivalent characteristic turbulence length scale. However, it is necessary to resolve which length scale, l, should be used, together with the inter-particle spacing λ, to decide a new hybrid gas turbulent dissipation scale. For gas–solid flow in a pipe, the pipe wall and the particles are the physical boundaries which restrict the gas turbulence. Consequently, l should be close to the pipe diameter.

Given all these conditions above, we adopt the model of Yoshizawa (1987) who suggested values for C_1 and C_2 of 0.043 and 1.84, respectively. Because the hybrid length scale l_h is a heuristical combination, this approach still requires further investigation.

The particle–gas fluctuation kinetic energy, k_{12} can be modelled by adopting the argument of Koch and Sangani (1999), who pointed out that this energy is related to the auto-correlation of the force felt by a representative particle. The correlation time is taken to be comparable to the collision interval t_2^c, and the force acting on a particle is comparable to the mean drag force, $\overline{\boldsymbol{F}}_\mathrm{drag}$. As a result, this particle–gas fluctuation kinetic energy can be expressed as

$$k_{12} = \frac{\beta_0 U_\mathrm{r}^2 t_2^\mathrm{c}}{\rho_2 \varepsilon_2}. \tag{23}$$

Because there is no satisfactory work available on flows with finite particle Reynolds number, Eq. (23) is an algebraic simplification of a complex physical situation. It may be taken as a "right order of magnitude" approach.

2.3. Fully developed steady flow

There is still no universal turbulent model, even for a single gas flow. For a multiple-phase system, the particle–gas turbulence interaction also needs further investigation. In order to incorporate the hybrid length scale readily into the gas turbulence modification model, and also to avoid the many uncertain empirical parameters in a two-equation model, we start from a one-equation turbulence model here. In order to identify the difference between the different turbulent modification models of Eqs. (15) and (16) easily, we apply the above models to a fully developed, steady, relatively dilute gas–solid flow in a vertical pipe. The governing equations then become:

Solid phase:

- Momentum equation

$$-\varepsilon_2 \frac{\partial P}{\partial z} - \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{2rz}) + \overline{F}_{\mathrm{drag},z} - \varepsilon_2 \rho_2 g_z = 0 \quad \text{(axial direction)}, \tag{24}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{2rr}) - \frac{\tau_{2\theta\theta}}{r} = 0 \quad \text{(radial direction)}. \tag{25}$$

- Fluctuational energy equation

$$-\frac{1}{r}\frac{\partial}{\partial r}(rq_\mathrm{r}) - \tau_{2rz}\frac{\partial u}{\partial r} - \beta_0(3T - \langle \boldsymbol{u}' \cdot \boldsymbol{v}' \rangle) - I = 0. \tag{26}$$

Gas phase:

- Momentum equation

$$-\varepsilon_1 \frac{\partial P}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}\left[r(\mu + \mu_\mathrm{t})\frac{\partial v}{\partial r}\right] - \overline{F}_{\mathrm{drag},z} = 0. \tag{27}$$

- Turbulent kinetic energy equations

$$\frac{1}{r}\frac{\partial}{\partial r}\left[\sigma_k r \frac{\partial k_1}{\partial r}\right] - \varepsilon_1 \tau_{1rz}\frac{1}{r}\frac{\partial}{\partial r}(rv) + \beta_0(\langle \boldsymbol{u}' \cdot \boldsymbol{v}' \rangle - 2k_1) - \varepsilon_1 \rho_1 \varepsilon = 0, \tag{28}$$

or

$$\frac{1}{r}\frac{\partial}{\partial r}\left[\sigma_k r\frac{\partial k_1}{\partial r}\right] - \varepsilon_1\tau_{1rz}\frac{1}{r}\frac{\partial}{\partial r}(rv) + \beta_0|u-v|^2 + \beta_0(3T - \langle \boldsymbol{u}'\cdot\boldsymbol{v}'\rangle) - \varepsilon_1\rho_1\varepsilon = 0, \tag{29}$$

where u and v are the mean axial velocities for the particulate phase and gas phase, respectively, and subscripts r and z represent the radial and axial directions, respectively.

3. Boundary conditions

For a gas–solid flow, the particle diameter may not be negligible when compared to the width of the gas viscous boundary layer. Therefore, when we set the boundary conditions at the wall for the solid phase, we establish momentum and energy balance at a thin layer of particles, the thickness of which is the same order as the gas viscous layer. The gas phase boundary condition should also be set on the same thin layer, as suggested by Sinclair and Jackson (1989). As a result, it may not be appropriate to use a low Reynolds number model and set non-slip boundary condition at the wall.

Here, we use a high Reynolds number model, adopt wall functions, and set the radial gradient of gas turbulent kinetic energy at the wall to zero, which means the generation and dissipation of the turbulent kinetic energy is nearly the same. The boundary conditions for the gas phase are not set at the wall, but at a distance comparable to the particle diameter.

The collisions between particles and the wall play a significant role in the flow in a small-diameter pipe, especially for high-inertial particles. Therefore, properly defining the boundary conditions for the particulate phase is essential in the simulation of gas–solid flows. Jenkins (1992), and Jenkins and Louge (1997) rigorously derived a set of boundary conditions for the solid phase which are averaged from the impulse equations for the collision of a particle with a plane wall. Their model is empirical-parameter free and is based on three constants: the coefficient of friction, and the normal and tangential coefficients of restitution. However, in practice, the normal coefficient of restitution is strongly velocity-dependent, and the range of sliding and non-sliding collisions that are likely to occur leads to a discontinuity in the integral.

The competing boundary conditions of Johnson et al. (1990) adopt an alternative approach. Although not mathematically rigorous, employing a specularity coefficient (which is an empirical "tunable" constant) may capture both the averaged influences of the geometry of the curved pipe wall and the variety of particle–wall collisions under one parameter. We therefore use the boundary conditions given in Zhang and Reese (2000), which are modified from the work of Johnson et al. Although the profiles of granular temperature are sensitive to the specularity coefficient in the calculations, this parameter does give some necessary flexibility in the boundary conditions. To ensure comparability of our results, once this specularity coefficient is adjusted in the numerical simulation of one operational condition it is kept the same for all our subsequent simulations.

In the centreline of the pipe, axi-symmetric conditions are applied, i.e.

$$\frac{\partial u}{\partial r} = 0, \quad \frac{\partial v}{\partial r} = 0, \quad \frac{\partial \varepsilon_2}{\partial r} = 0, \quad \frac{\partial T}{\partial r} = 0, \quad \frac{\partial k_1}{\partial r} = 0. \tag{30}$$

4. Results and discussion

There are few non-intrusive experimental measurements of particle–gas turbulence interaction, even for relatively simple flows such as up-flow in a vertical pipe. Tsuji et al. (1984) reported their laser Doppler velocimetry measurements of fully developed steady gas–solid flows in a vertical pipe. In 1993, Tsuji revisited the data and his profiles of the axial fluctuational velocity of the solid phase were cited in the work of Bolio et al. (1995). Currently, these experimental data produced by Tsuji et al. are the most comprehensive, involving differently sized particles, different mass loading ratios and various superficial gas velocities. Moreover, not only the fluctuational velocities of both phases were measured, but also their mean velocity. In order to examine our model thoroughly, we will compare our simulation results with these data.

Tsuji et al. (1984) used polystyrene spheres with a density of 1020 kg/m^3 and diameters of 200, 500, 1000, and 3000 μm. The internal diameter of the pipe was 30.5 mm. The results were reported for different particle-to-gas mass flow rates, or mass loading ratios, m, which are defined via

$$m = \frac{\int_0^R \rho_2 \varepsilon_2 u r \, \mathrm{d}r}{\int_0^R \rho_1 \varepsilon_1 v r \, \mathrm{d}r}. \tag{31}$$

In our calculation, we take the gas density to be 1.2 kg/m^3. Tsuji et al. conducted experiments up to $m = 4.2$.

It is difficult to find suitable coefficients of restitution for particle–particle, e, and particle–wall, e_w collisons. Following Louge et al. (1991) and Bolio et al. (1995), we adopt $e = 0.9$ and $e_w = 0.75$. The coefficient of restitution depends on the impact velocity, so it should not be treated as a constant. Although this is the most delicate parameter in the kinetic theory of granular flow, reasonable estimated constant values of e and e_w can give at least a first-order prediction when the collisions are nearly elastic. In the final part of this paper, we give a sensitivity analysis of our results to this coefficient of restitution.

Because there are no universally accepted models for a gas–solid two phase flow, even for simple fully developed steady vertical pipe flow, we compare the results of solving different turbulent kinetic energy balance equations, i.e. Eqs. (28) and (29). The turbulence length scales will be l, as in the pure gas flow, and l_h as given in Eq. (21). Moreover, the impact of the interstitial turbulent gas on the constitutive equations for the solid phase will also be investigated. For the sake of clarity, we identify four models for investigation:

Model 1. The gas turbulent kinetic energy equation is given by Eq. (29) and the turbulent characteristic length scale is l_h.

Model 2. The gas turbulent kinetic energy equation is given by Eq. (28) and the turbulent characteristic length scale is l.

Model 3. The gas turbulent kinetic energy equation is given by Eq. (28) but the turbulent characteristic length scale is l_h.

Model 4. As Model 1, but the interstitial gas influence on the random motion of particles is not accounted for.

The effect of the interstitial gas turbulence on the constitutive equations of the solid phase is incorporated in the first three models.

For fully developed steady flow in an axi-symmetric pipe, the governing equations reduce to four coupled second-order, non-linear ordinary differential equations, and one algebraic equation, as given in Eqs. (24)–(29). Because the solution is very sensitive to the initial guess, the four coupled non-linear equations are decoupled and linearised. A finite difference method is then used to solve these ordinary differential equations, and convergence is rapidly reached by using an over-relaxation method.

Louge et al. (1991) first adopted the kinetic theory of dry granular flow with a one-equation turbulence model and used the same turbulence length scale as in pure gas flow. They produced results which are in quite good agreement with the experimental data of Tsuji et al. (1984). From our Fig. 1, we see that Model 2 cannot predict the observation that gas turbulent kinetic

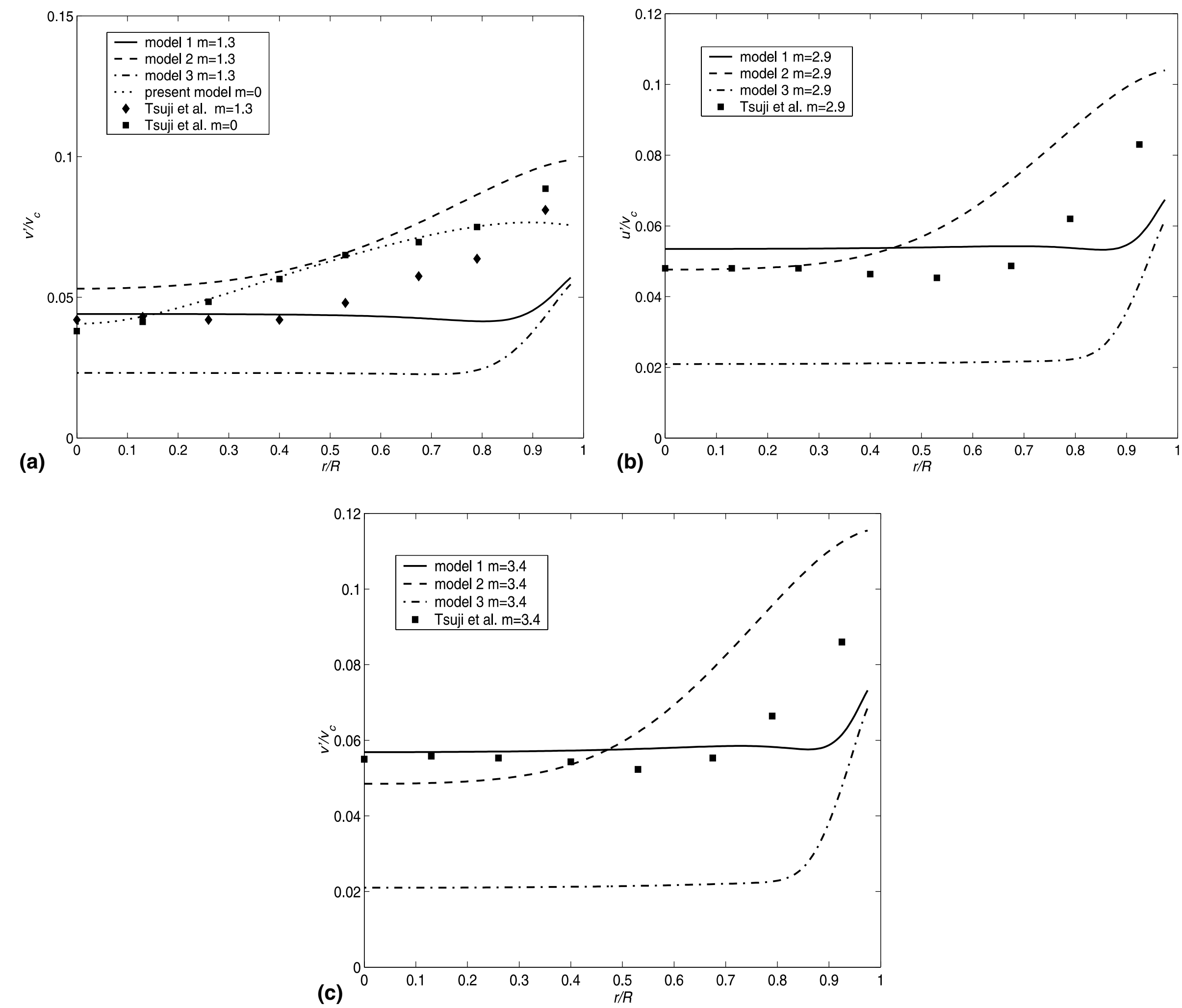


Fig. 1. Comparison of gas fluctuational velocity profiles for: (a) mass loading ratio, $m = 1.3$ or 0 with gas pipe-axis velocity, $v_c = 13.3$ m/s; (b) $m = 2.9$ with $v_c = 11.4$ m/s; (c) $m = 3.4$ with $v_c = 10.7$ m/s. The particles are 500 μm diameter.

energy is enhanced with increasing mass loading ratios, m. Model 3, with a modified turbulent length scale, evidently under-estimates the axial fluctuational velocity and also fails to predict the enhancement of turbulent kinetic energy. Bolio et al. (1995) adopted Eq. (28) for a k–ε model and added an extra term to account for the produced wake effect on turbulence enhancement. However, our Model 1 tests whether the work done by the drag force is responsible for the enhancement of turbulent intensity, and whether the additional bounding inter-particle space is responsible for the attenuation of the turbulent intensity. These results are in reasonable agreement with the experimental data. More importantly, Model 1 predicts the enhancement of gas turbulence with increasing mass loading ratio, and the turbulent intensity increase in the core region and decrease near the wall. Our model assumes the flow is isotropic, but actual gas–

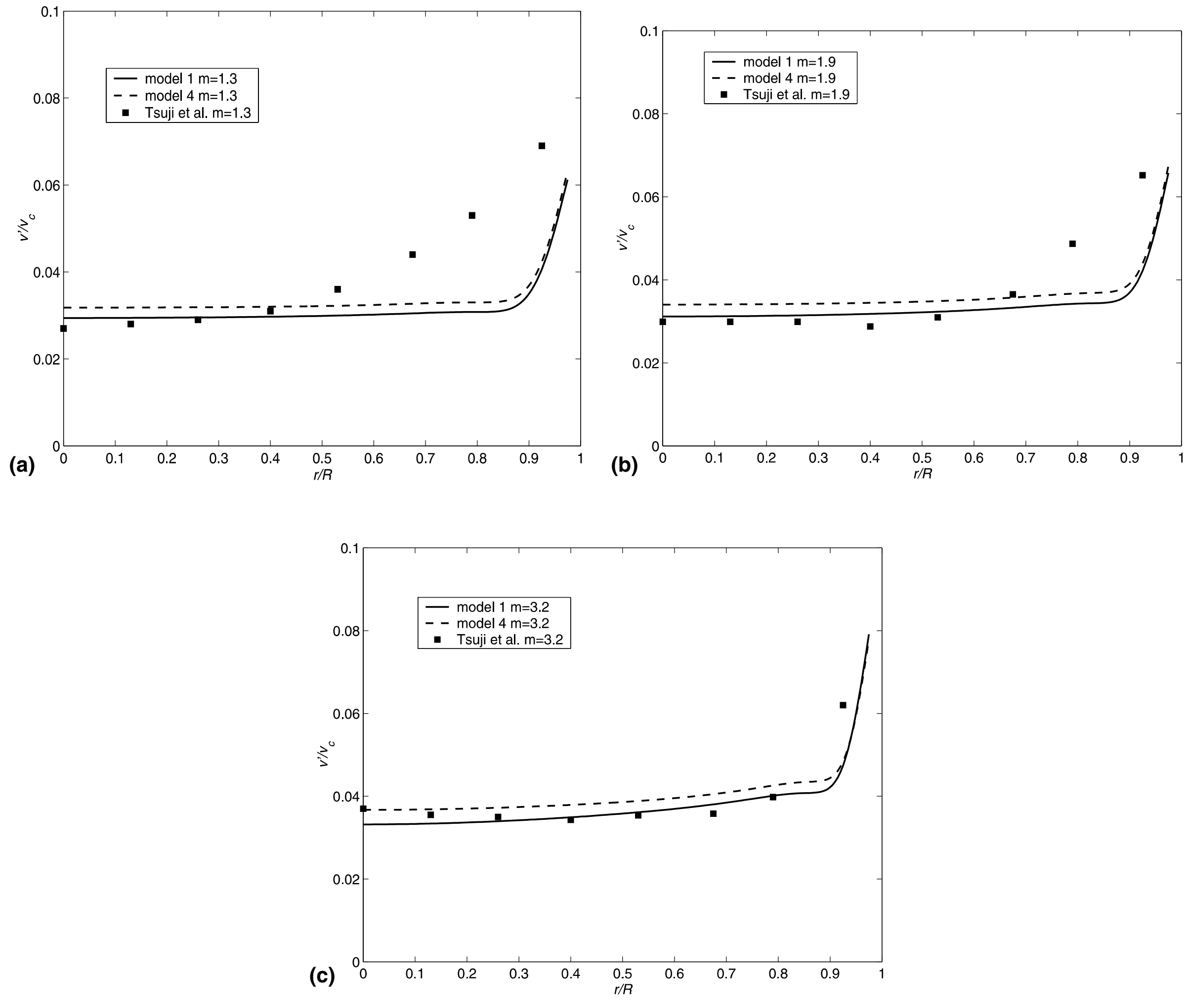


Fig. 2. Comparison of gas fluctuational velocity profiles for: (a) mass loading ratio, $m = 1.3$ with gas pipe-axis velocity, $v_c = 12.8$ m/s; (b) $m = 1.9$ with $v_c = 11.9$ m/s; (c) $m = 3.2$ with $v_c = 10.8$ m/s. The particles are 200 μm diameter.

solid flows are not isotropic, particularly near the wall. Consequently, we cannot reasonably expect our predicted results to be in perfect agreement with the measured data, especially near the wall. Both experimental measurements and our numerical simulations reveal the phenomenon that the profiles of gas turbulent intensity are more flattened due to the presence of the particles.

From the comparisons of the results in Fig. 1, we now focus on Model 1 and investigate the influence of the gas turbulence on the microscopic motion of the particle phase. In Figs. 2 and 3, the particles are 200 μm diameter, and mass loading ratios are $m = 1.0$, 1.3, 3.2 and 4.2, respectively. The profiles of the gas fluctuational velocity are in reasonable agreement with the measurements. The flattened core regimes are also predicted, although in the near-wall regime our

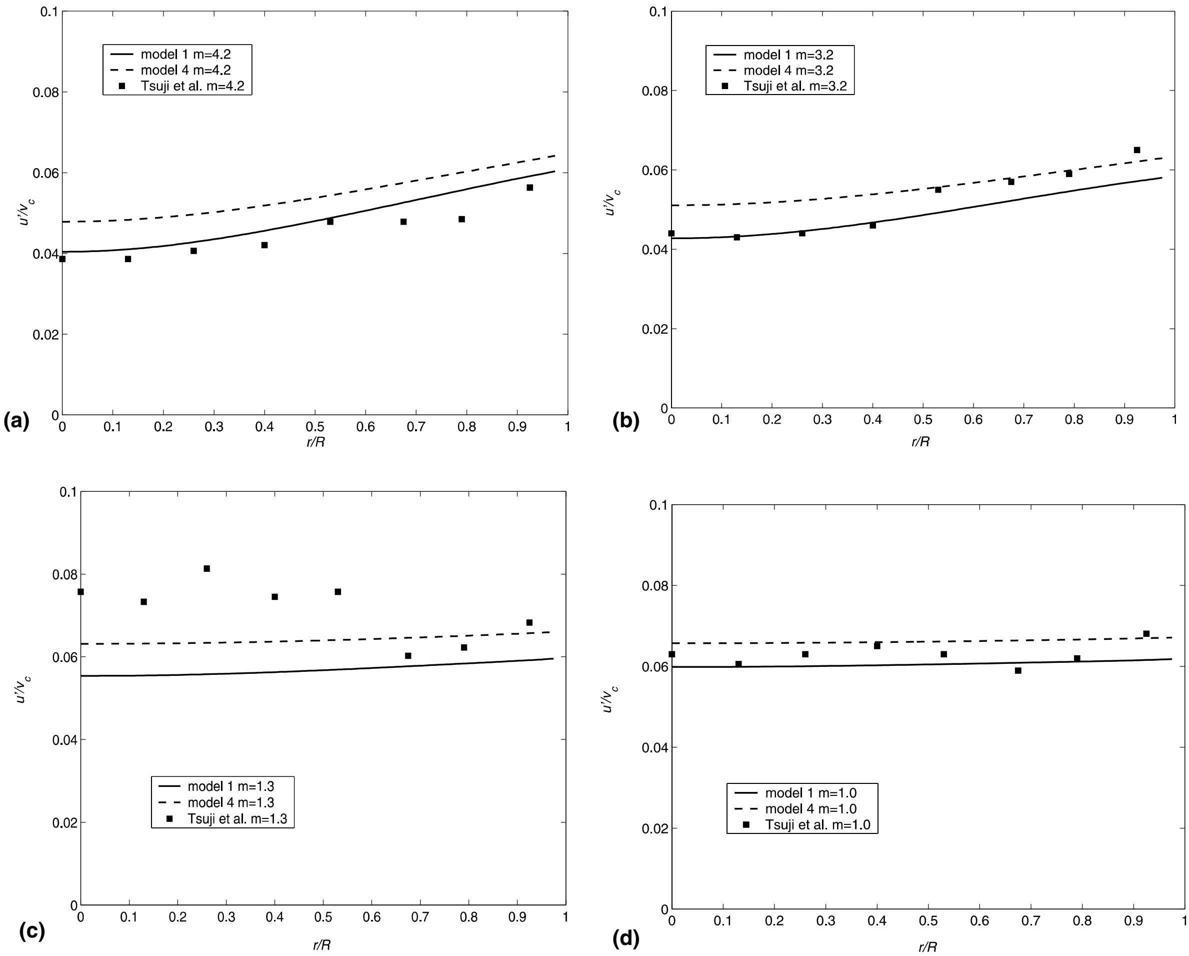


Fig. 3. Comparison of solid fluctuational velocity profiles for: (a) mass loading ratio, $m = 4.2$ with gas pipe-axis velocity, $v_c = 14.6$ m/s; (b) $m = 3.2$ with $v_c = 10.8$ m/s; (c) $m = 1.3$ with $v_c = 12.8$ m/s; (d) $m = 1.0$ with $v_c = 18.9$ m/s. The particles are 200 μm diameter.

models under-predict the turbulent intensity. We can distinguish the difference between Models 1 and 4: Model 4 predicts slightly greater fluctuations than Model 1. As Louge et al. (1991) say, the particle relaxation time is much larger than the characteristic time-scale of the eddies so the particle does not follow the gas turbulence. But this hydrodynamic relaxation time is at most one order larger than the particle collision time, or nearly the same order in these cases, so the gas flow affects the particles' random motion, as seen in Fig. 2. In Fig. 3, the simulation results of Model 1 for the profiles of the axial fluctuational velocity of the solid phase generally fit well with the measurements. The fluctuations are found to increase when the mass load decreases, except for the mass loading ratios of 1.3 and 1.0. We also see the slight difference between the results of our two models.

Figs. 4 and 5 compare the fluctuational velocity of the gas phase containing 500 and 1000 μm particle spheres at different mass loading ratios. Generally, the gas turbulent intensity is enhanced by the large particles and the turbulent intensity profiles flattened. Also, for these larger

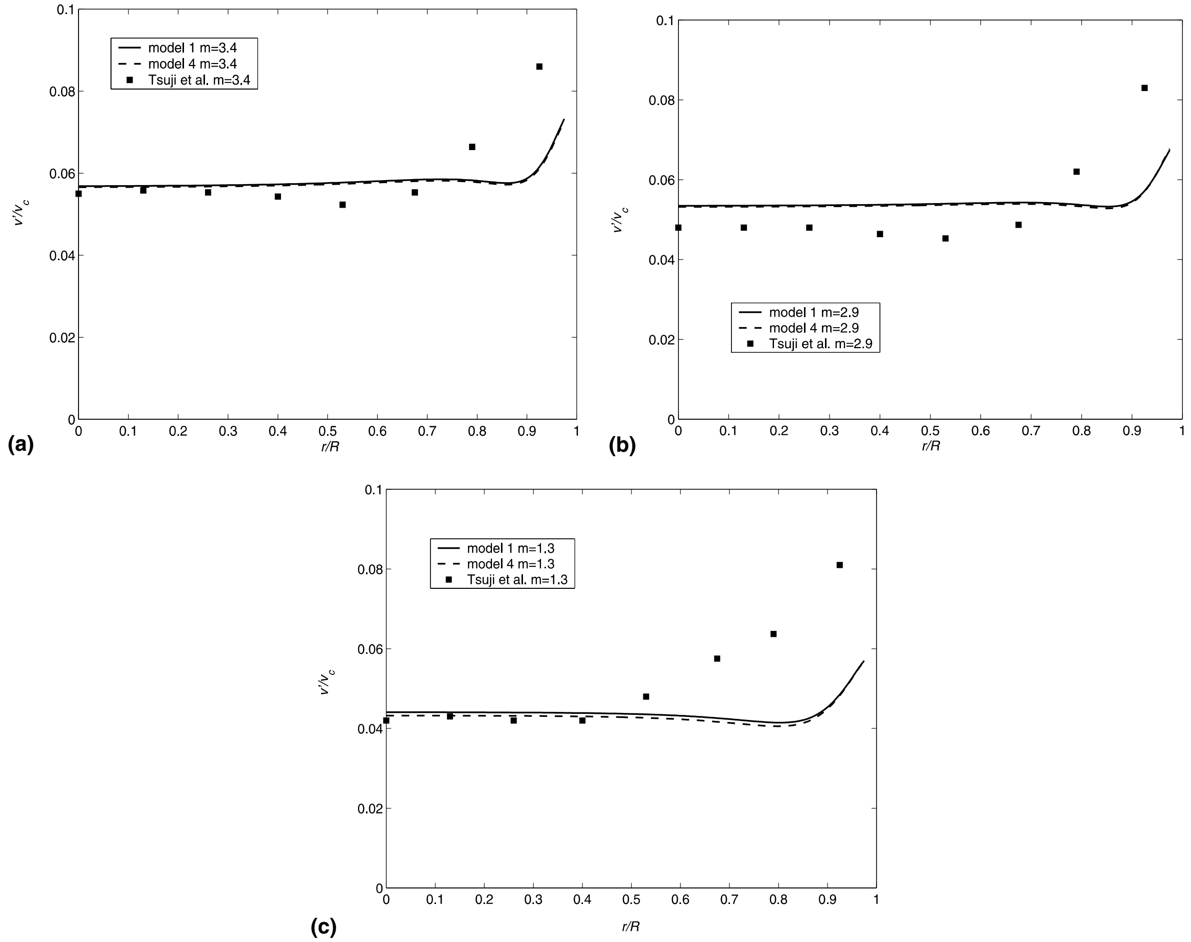
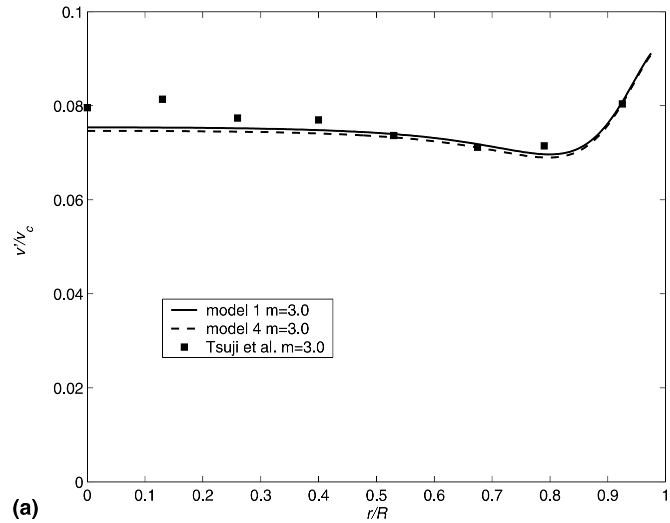


Fig. 4. Comparison of gas fluctuational velocity profiles for: (a) mass loading ratio, $m = 3.4$ with gas pipe-axis velocity, $v_c = 10.7$ m/s; (b) $m = 2.9$ with $v_c = 11.4$ m/s; (c) $m = 1.3$ with $v_c = 13.3$ m/s. The particles are 500 μm diameter.



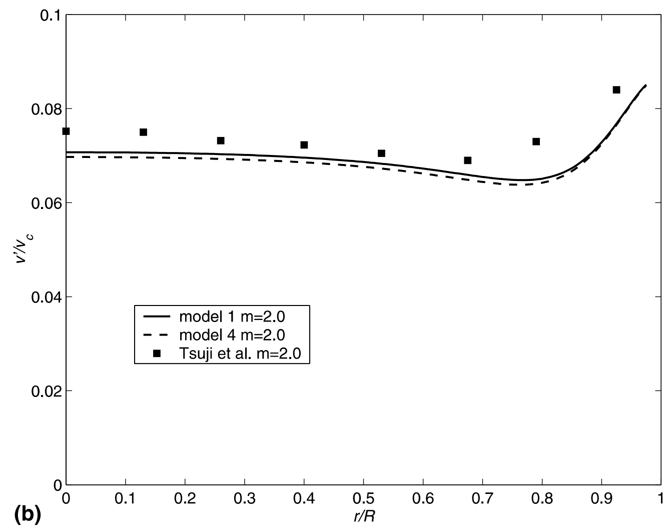


Fig. 5. Comparison of gas fluctuational velocity profiles for: (a) mass loading ratio, $m = 3.0$ with gas pipe-axis velocity, $v_c = 13.2$ m/s; (b) $m = 2.0$ with $v_c = 12.8$ m/s. The particles are 1000 μm diameter.

particles, whether or not we include the influence of the gas turbulence on modelling the particle phase constitutive behaviour has very little impact on the flows. The particle stress has two components: the collisional part which is proportional to the particle diameter, and the turbulent part which relates to the ratio of the particle relaxation time, t_{12}^{x}, and the collisional interval, t_2^c. We find that t_2^c is nearly one order smaller than t_{12}^{x}, which means that the random motion of particles is mainly controlled by the collisions but the gas turbulence influence on the particle turbulent stress should also be considered. However, this can only have a noticeable

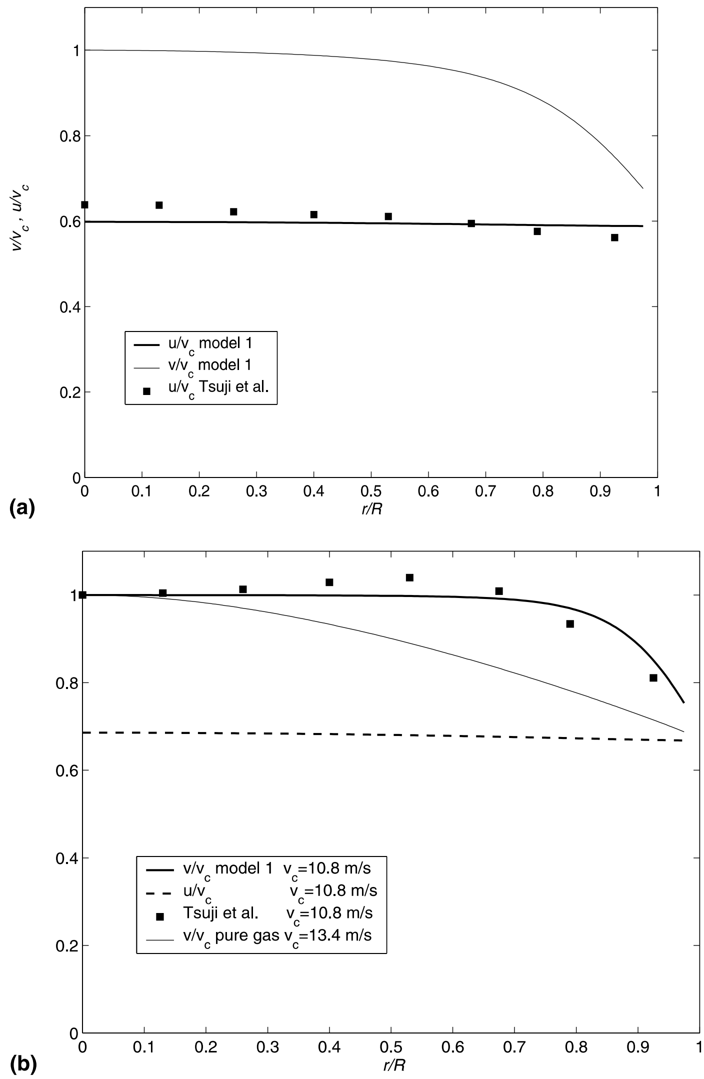


Fig. 6. Comparison of velocity profiles of the gas and solid phases for: (a) mass loading ratio, $m = 1.1$ with gas pipe-axis velocity, $v_c = 9.65$ m/s; (b) $m = 3.4$ with $v_c = 10.7$ m/s. The particles are 500 μm diameter.

effect on the total stress if the particles are small. As a result, the effect can be neglected for the larger particles here.

Comparisons of the gas and the particle axial velocities are given in Figs. 6 and 7. Because there are no simultaneous measurements of the velocity profiles of the gas and the particles, we can only compare them for different cases. Evidently, the particle phase has a larger slip velocity at the wall. For smaller particles, such as 200 μm in Fig. 7, the gas velocity is lower than the particle velocity close to the wall. This phenomenon has been experimentally observed by Tsuji et al. (1984) and Lee and Durst (1982). It may be interpreted in terms of the particle shear stress

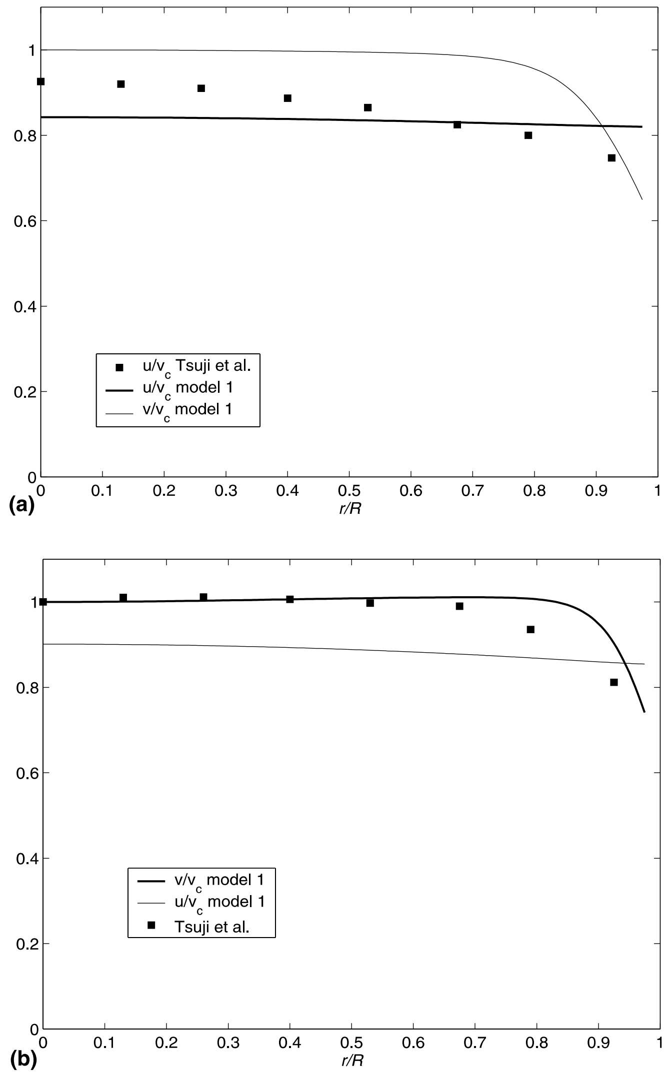


Fig. 7. Comparison of velocity profiles of the gas and solid phases for: (a) mass loading ratio, $m = 1.0$ with gas pipe-axis velocity, $v_c = 18.9$ m/s; (b) $m = 3.2$ with $v_c = 10.8$ m/s. The particles are 200 μm diameter.

(Louge et al., 1991). The gas velocity profiles are found to be more flattened than for pure gas flow.

Finally, we examine the sensitivity of our results to the particle–particle coefficient of restitution. As discussed in Louge et al. (1991), the particle–particle coefficient of restitution $e = 0.9$ could be a close estimate. Although our model does not suffer from over-sensitivity to the restitution coefficient, as the work of Sinclair and Jackson (1989) and Pita and Sundaresan (1991, 1993) did for their denser cases, this coefficient still plays an important role in the flow, as can be seen in Fig. 8. Here, we keep e_w as 0.75 throughout and change e only. We can see that the fluctuational velocity of the

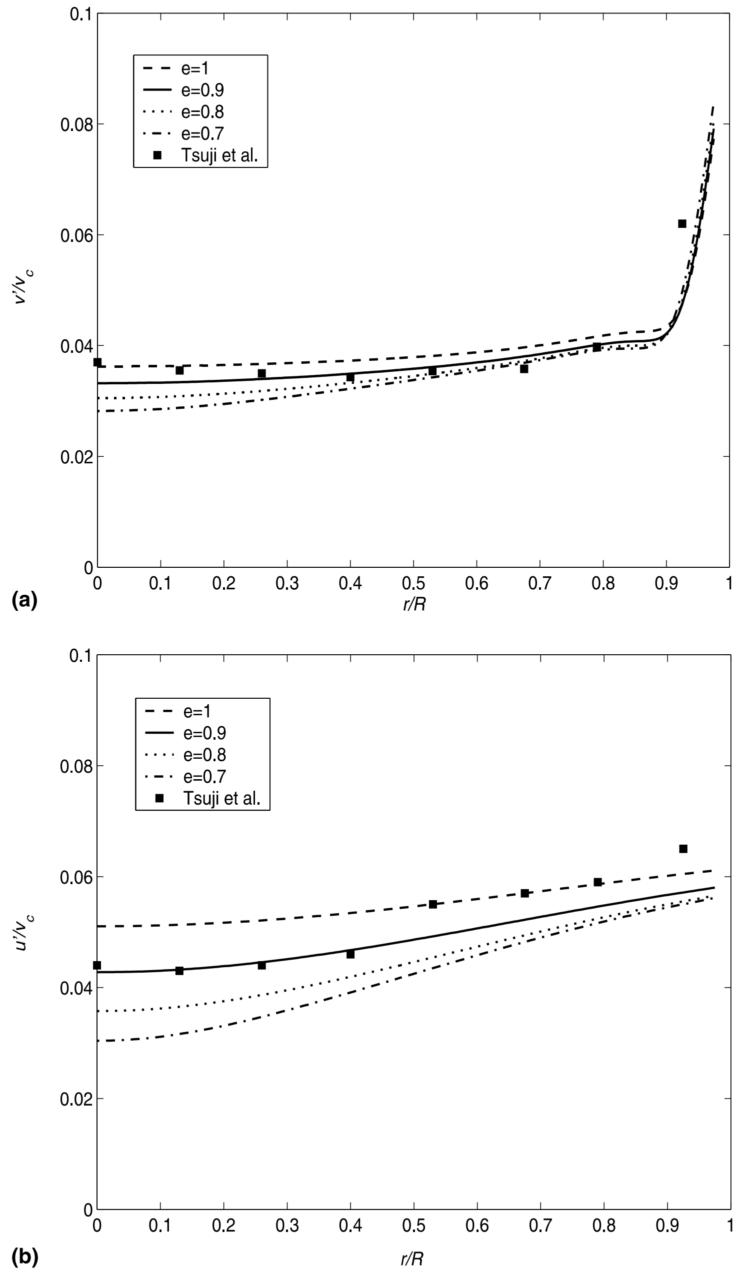


Fig. 8. Comparison of fluctuational velocity profiles for: (a) the gas phase; (b) the solid phase. The particles are 200 μm diameter with differing particle–particle restitution coefficients. Mass loading ratio, $m = 3.2$ with gas pipe-axis velocity, $v_c = 10.8$ m/s.

particle phase at the axis varies by up to 40% between $e = 1$ and $e = 0.7$ while it varies about 25% for the gas phase. However, if our estimated coefficient of restitution is close to the real system-averaged one, the deviation of the results could still be acceptable.

5. Conclusions

The gas turbulence has an impact on the microscopic motion of the particles which cannot be neglected for smaller particles in relatively dilute flows. However, it may have a negligible effect on larger particles, even though the particle volume fraction is small. The turbulence modulation model of Crowe and his colleagues can predict the gas turbulence modification due to the presence of the particles reasonably well, and displays few problems of undue sensitivity to the particle–particle restitution coefficient. It is our opinion that it should be investigated further for its applications to granular-gas flows.

Acknowledgements

The authors are grateful to Professor Crowe for offering copies of his colleagues' and his recent works and for the helpful discussions. We also thank the referees for their useful comments, which helped improve several aspects of this paper.

Appendix A

From the work of Peirano and Leckner (1998), we adopt the following expressions for the essential particle and gas flow properties. The collisional viscosity is given by

$$\nu_2^{\mathrm{c}} = \frac{4}{5}\varepsilon_2\chi(1+e)\left(\nu_2^{\mathrm{t}} + d\sqrt{\frac{T}{\pi}}\right) \tag{A.1}$$

and the turbulent viscosity by

$$\nu_2^{\mathrm{t}} = \left(\frac{2}{3}\frac{t_{12}^{\mathrm{t}}}{t_{12}^{x}}k_{12} + (1+\varepsilon_2\chi A)T\right)\Big/\left(\frac{2}{t_{12}^{x}} + \frac{B}{t_2^{\mathrm{c}}}\right), \tag{A.2}$$

where

$$A = \frac{2}{5}(1+e)(3e-1) \quad \text{and} \quad B = \frac{1}{5}(1+e)(3-e).$$

The interaction time between particle motion and gas fluctuations is

$$t_{12}^{\mathrm{t}} = t_1^{\mathrm{t}}(1 + C_\beta\zeta_{\mathrm{r}})^{-1/2}, \tag{A.3}$$

where $\zeta_{\mathrm{r}} = 3|U_{\mathrm{r}}|^2/2k_1$; C_β is a constant; and the characteristic time-scale of the eddy is

$$t_1^{\mathrm{t}} = C_\mu k_1/\varepsilon, \tag{A.4}$$

where C_μ is a constant. The particle relaxation time is

$$t_{12}^{x} = \frac{4d\rho_2}{3\rho_1\langle C_{\mathrm{D}}\rangle\langle|u_{\mathrm{r}}|\rangle} \tag{A.5}$$

and the particle collision time is

$$t_2^c = \frac{d}{24\varepsilon_2\chi}\sqrt{\frac{\pi}{T}}, \tag{A.6}$$

where χ is the radial distribution function which, in this paper, is described by the expression of Lun and Savage (1987), viz.

$$\chi = \left[1 - \frac{\varepsilon_2}{\varepsilon_{2m}}\right]^{-2.5\varepsilon_{2m}}, \tag{A.7}$$

where ε_{2m} represents the maximum possible particle fraction of the system, i.e. 0.645 in the calculation. The collisional diffusion coefficient is given by

$$k_2^c = \varepsilon_2\chi(1+e)\left(\frac{6}{5}k_2^t + \frac{4}{3}d\sqrt{\frac{T}{\pi}}\right) \tag{A.8}$$

and the turbulent diffusion coefficient by

$$k_2^t = \left[\frac{3t_{12}^t}{5t_{12}^x}k_{12} + (1+\varepsilon_2\chi C)T\right] \Big/ \left(\frac{9}{5t_{12}^x} + \frac{D}{t_2^c}\right), \tag{A.9}$$

where $C = 3(1+e)^2(2e-1)/5$ and $D = (1+e)(49-33e)/100$.

References

Abu-Zaid, S., Ahmadi, G., 1990. Simple kinetic model for rapid granular flows including frictional losses. J. Eng. Mech. ASCE 116, 379–389.

Anderson, T.B., Jackson, R., 1967. Fluid mechanical description of fluidized beds: comparison with theory and experiment. I&EC Fund 6, 527–539.

Bagnold, R.A., 1954. Experiments on a gravity-free dispersion of large solid particles in a Newtonian fluid under shear. Proc. R. Soc. London A 225, 49–63.

Berlemont, A., Desjonqueres, P., Gouesbet, G., 1990. Particle Lagrangian simulation in turbulent flows. Int. J. Multiphase Flow 16, 19–34.

Bolio, E.J., Sinclair, J.L., 1995. Gas turbulence modulation in the pneumatic conveying of massive particles in vertical tubes. Int. J. Multiphase Flow 21, 985–1001.

Bolio, E., Yasuna, J., Sinclair, J., 1995. Dilute turbulent gas–solid flow in risers with particle interactions. AIChE J. 41, 1375–1388.

Campbell, C.S., 1990. Rapid granular flows. Annu. Rev. Fluid Mech. 22, 57–92.

Chen, P.E., Wood, C.E., 1985. A turbulence closure model for dilute gas–particle flows. Can. J. Chem. Eng. 63, 349–360.

Crowe, C.T. 1997. Recent advances in modelling of turbulent gas particle flows. In: ISMNP' 97, International Symposium on Multiphase Fluid, Non-Newtonian Fluid and Physico Chemical Fluid Flows.

Crowe, C.T., Gillandt, I. 1998. Turbulence modulation of fluid-particle flows – a basic approach. In: Third International Conference on Multiphase Flows, ICMF'98, Lyon, France.

Crowe, C.T., 2000. On models for turbulence modulation in fluid-particle flows. Int. J. Multiphase Flow 26, 719–727.

Crowe, C.T., Wang, P. 2000. Towards a universal model for carrier-phase turbulence in dispersed phase flows. In: Proceedings of 2000 ASME FED Summer Meeting, FEDSM2000-11132, Boston, USA.

Elghobashi, S.E., 1994. On predicting particle-laden turbulent flows. Appl. Sci. Res. 52, 309–329.

Gidaspow, D., 1994. Multiphase Flow and Fluidization. Academic Press, London.

Jenkins, J.T., 1992. Boundary conditions for rapid granular flow: flat, frictional walls. J. Appl. Mech. 59, 120–127.

Jenkins, J.T., Louge, M.Y., 1997. On the flux of fluctuation energy in a collisional grain flow at a flat, frictional wall. Phys. Fluids 9, 2835–2840.

Jenkins, J.T., Richman, M.W., 1985. Grad's 13-moment system for a dense gas of inelatic spheres. Arch. Rat. Mech. Anal. 87, 3485–3494.

Jenkins, J.T., Savage, S.B., 1983. A theory for the rapid flow of identical, smooth, nearly elastic particles. J. Fluid Mech. 130, 187–202.

Johnson, P.C., Nott, P., Jackson, R., 1990. Frictional–collisional equations of motion for particulate flows and their application to chutes. J. Fluid Mech. 210, 501–535.

Kenning, V.M., Crowe, C.T., 1997. Effect of particle on carrier phase turbulence in gas–particle flows. Int. J. Multiphase Flow 23, 403–408.

Koch, D., Sangani, A.S., 1999. Particle pressure and marginal stability limits for a homogeneous monodisperse gas-fluidized bed: kinetic theory and numerical simulations. J. Fluid Mech. 400, 229–263.

Launder, B.E., Spalding, D.B., 1972. Lectures in Mathematical Models of Turbulence. Academic Press, London.

Lee, S.L., Durst, F., 1982. On the motion of particles in turbulent duct flows. Int. J. Multiphase Flow 8, 125–146.

Littman, H., Morgan III, M.H., Paccione, J.D., 1996. A pseudo-Stokes representation of effective drag coefficient for large particles entrained in a turbulent air stream. Powder Technol. 87, 169–173.

Louge, M.Y., Mastorakos, E., Jenkins, J.K., 1991. The role of particle collisions in pneumatic transport. J. Fluid Mech. 231, 345–359.

Lun, C.K.K., Savage, S.B., 1987. A simple kinetic theory for granular flow of rough, inelastic, spherical particles. Trans. ASME J. Appl. Mech. 54, 47–53.

Lun, C.K.K., Savage, S.B., Jeffrey, D.J., Chepurniy, N., 1984. Kinetic theory for granular flow: inelastic particles in Couette flow and slightly inelastic particles in a general flowfield. J. Fluid Mech. 140, 223–256.

Mathiesen, V., Solberg, T., Hjertager, B.H., 2000. Predictions of gas/particle flow with an Eulerian model including a realistic particle size distribution. Powder Technol. 112, 34–45.

Neri, A., Gidaspow, D., 2000. Riser hydrodynamics: simulation using kinetic theory. AIChE J. 46 (1), 52–67.

Nieuwland, J.J., Annaland, M., Kuipers, J.A.M., Swaaij, W.P.M., 1996. Hydrodynamic modelling of gas/particle flows in riser reactors. AIChE J. 42, 1569–1582.

Ogawa, S., Umemura, A., Oshima, N., 1980. On the equations of fully fluidized granular materials. J. Appl. Math. Phys. (ZAMP) 31, 483–493.

Peirano, E., Leckner, B., 1998. Fundamentals of turbulent gas–solid flows applied to circulating fluidized bed combustion. Prog. Energy Combust. Sci. 24, 259–296.

Pita, J., Sundaresan, S., 1991. Gas–solid flow in vertical tubes. AIChE J. 37, 1009–1018.

Pita, J., Sundaresan, S., 1993. Developing flow of a gas–particle mixture in a vertical riser. AIChE J. 39, 541–552.

Rodi, W., 1993. Turbulence Models and their Application in Hydraulics. A.A. Balkema, Rotterdam, Netherlands.

Savage, S.B., 1984. The mechanics of rapid granular flows. Adv. Appl. Mech. 24, 289–366.

Savage, S.B., Jeffrey, D.J., 1981. The stress tensor in a granular flow at high shear rates. J. Fluid Mech. 110, 255–272.

Sinclair, J.L., Jackson, R., 1989. Gas–particle flow in a vertical pipe with particle–particle interactions. AIChE J. 35 (9), 1473–1486.

Tsuji, Y., Morikawa, Y., Shiomi, H., 1984. LDV measurements of an air–solid two-phase flow in a vertical pipe. J. Fluid Mech. 139, 417–434.

Tsuji, Y. 1993. Private communication cited in Bolio, E.J., Sinclair, J.L. 1995. Gas turbulence modulation in the pneumatic conveying of massive particles in vertical tubes. Int. J. Multiphase Flow 21, 985–1001.

Yarin, L.P., Hetsroni, G., 1994. Turbulence intensity in dilute two-phase flows. Int. J. Multiphase Flow 20, 27–44.

Yoshizawa, A., 1987. Statistical modelling of a transport equation for the kinetic energy dissipation rate. Phys. Fluids A 30, 628–631.

Yuan, Z., Michaelides, E., 1992. Turbulence modulation in particulate flows – a theoretical approach. Int. J. Multiphase Flow 18, 779–785.

Zhang, Y., Reese, J.M., 2000. The influence of the drag force due to the interstitial gas on granular flows down a chute. Int. J. Multiphase Flow 26, 2049–2072.